Mechanics: Motion in one dimension

Objectives

- Describe the concepts of position and a frame of reference/point of origin.
- Define distance and displacement.
- Calculate distance and displacement for one dimensional motion.
- Define average speed and velocity.
- Define instantaneous speed and velocity.
- Calculate speed and velocity for one dimensional motion.
- Learn how to do speed conversions.
- Define acceleration.
- Differentiate between positive acceleration and negative acceleration.
- Calculate acceleration for one dimensional motion.
- Draw graphs of $\vec{x}\ vs\ t$, $\vec{v}\ vs\ t$ and $\vec{a}\ vs\ t$, and describe motion for:
    - Stationary objects.
    - Uniform motion
    - Constant acceleration.
- Determine the velocity of an object from the gradient of an $\vec{x}\ vs\ t$ graph.
- Determine the instantaneous velocity of an object from the tangent on an $\vec{x}\ vs\ t$ graph.
- Determine the acceleration of an object from the gradient of a $\vec{v}\ vs\ t$ graph.
- Determine the displacement of an object by finding the area under a $\vec{v}\ vs\ t$ graph.
- Determine the velocity of an object by finding the area under an $\vec{a}\ vs\ t$ graph.

Introduction

Motion in one dimension is about how things move along a straight line. In this section we will learn how to describe motion in one dimension and how to draw and analyse motion graphs. There are three features of motion that we use to describe how an object moves, displacement, velocity and acceleration. All three are vectors and therefore have magnitude and direction.

- Displacement, $\vec{x}$, measured in meters (m).
- Velocity, $\vec{v}$, measured in meters per second (m.s^{-1}).
- Acceleration, $\vec{a}$, measured in meters per second squared (m.s^{-2}).

Position and frame of reference / point of origin

When studying motion, we need to consider the **position** of an object relative to a **frame of reference**, also known as the **point of origin**. For example, you ask your friend Sarah to deliver a chocolate bar to your little sister, Jenna in her classroom. You'd tell Sarah that Jenna sits two desks to the left from the classroom door. This means that the classroom door is the frame of reference or the point of origin. If you don't include this frame of reference, then you cannot describe properly to Sarah where your sister sits.

What is position? [D]

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What is a frame of reference? [D]

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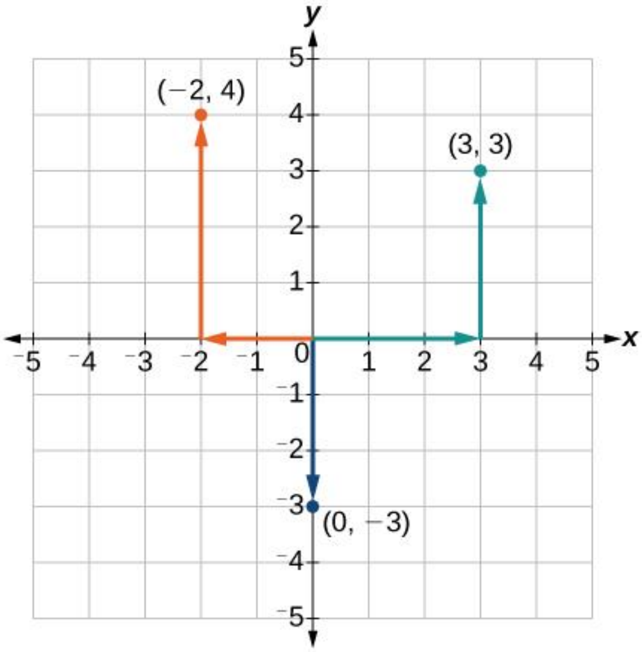
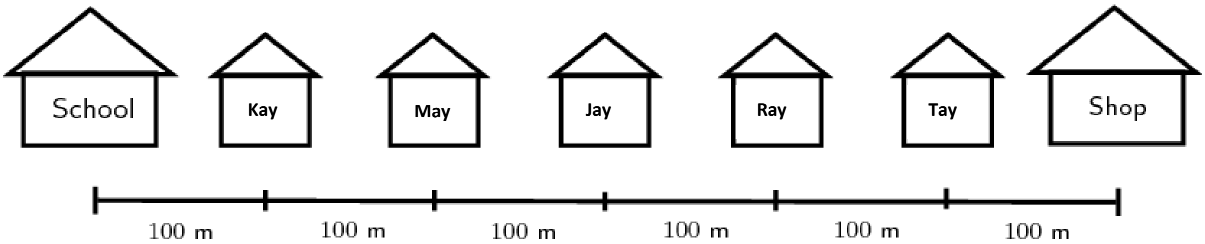
On a cartesian plane, there are x and y axes. The point of origin or frame of reference is where the x and y axes meet, at 0. The cartesian plane represents positive and negative x values as well as positive and negative y values. There are three positions shown in the graph below as (x,y) are:

- -2,4
- 3,3
- 0,-3



We do not need to use a cartesian plane with x and y coordinates. Any point of origin or frame of reference to describe position works. We need to state a frame of reference and a positive direction. Consider the diagram below:



- Assume that the point of origin is Jay's house and that left is the positive direction. This means that the school is 300 m from Jay's house in the positive direction (to the left). Tay's house is 200 m from Jay's house in the negative direction (to the right).
- Assume that the point of origin is May's house and that right is the positive direction. This means that the shop is 400 m from May's house in the positive direction (to the right). Kay's house is 100 m from May's house in the negative direction (to the left).

Distance and displacement

What is distance? [D]

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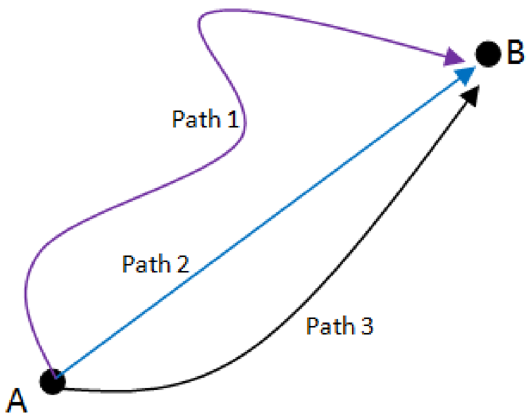
Distance is represented by D. The SI unit of distance is the meter (m). Distance is a scalar meaning that it has magnitude only.

Suppose a fly is trying to get from point A to point B. It might take any of the paths shown in the figure below. In each case, it would travel a different distance. Path 1 is the longest distance while path 2 is the shortest distance.



However, in each case, the fly started at point A and ended at point B. This brings us to the displacement. What is displacement? [D]

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Displacement is represented by $\Delta\vec{x}$ which means change (Δ) in position ($\vec{x}$). The SI unit of displacement is the meter (m). Displacement is a vector meaning that it has both magnitude and direction. In the figure above, although the fly travelled differently along all three paths, the displacement is the same in all three cases, because the starting and ending positions are the same in all three cases.

If the fly flies from A to B, and then back to A, it has travelled some distance based on the total length that it has travelled. However, its displacement will be zero because the starting point and ending point are the same.

The delta symbol Δ means change and <u>change</u> in anything is calculated as <u>final minus initial</u>. To represent the initial position we use the subscript ‘i’, e.g. $\vec{x}_i$. To represent the final position we use the subscript ‘f’, e.g. $\vec{x}_f$. Consider the formula for change in displacement. Change in displacement is equal to final position minus initial position:

$$\Delta\vec{x} = \vec{x}_f - \vec{x}_i$$

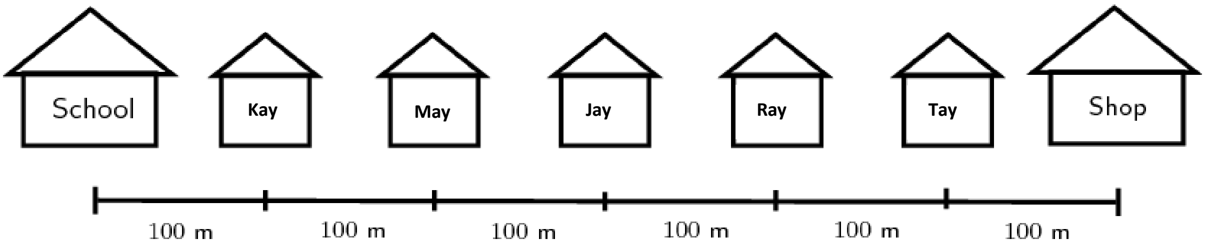
Displacement may be positive or negative depending on the direction that the object moves.

For the same motion, the distance and displacement may have different values. The table below summarises the differences between distance and displacement:

Distance	Displacement
Scalar	Vector
Always positive	May be positive or negative
Depends on the path taken	Does not depend on the path taken

Let's explore these concepts in more detail with the following examples:

Ray walks from his house to meet Kay at Kay's his house and together they walk to school



a) What distance did Ray cover?

Ray walked 300 m from his house to Kay's house and another 100 m to school.
$D = 300 + 100 = 400\ m$

b) What is Ray's displacement if Ray's house is the reference point and towards the school is the positive direction?

Ray's initial position is his house (the reference point) so it is 0 m.
Ray's final position is the school. The school is 400 m away from Ray's house (the reference point) in the positive direction, so it is +400 m.

$\Delta\vec{x} = \vec{x}_f - \vec{x}_i$
$\Delta\vec{x} = 400 - 0$
$\Delta\vec{x} = 400\ m$ towards the school (the positive answer means a positive direction, which is towards the school)

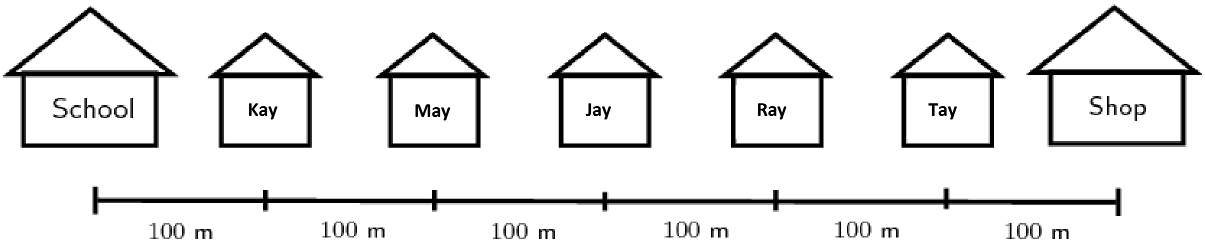
c) What is Ray's displacement if the shop is the reference point and towards the shop is the positive direction?

Ray's initial position is his house. Ray's house is 200 m away from the shop (the reference point) in the negative direction so it is –200 m.
Ray's final position is the school. The school is 600 m away from the shop (the reference point) in the negative direction, so it is –600 m.

$\Delta\vec{x} = \vec{x}_f - \vec{x}_i$
$\Delta\vec{x} = -600 - (-200)$
$\Delta\vec{x} = -400$
$\Delta\vec{x} = 400\ m$ towards the school (the negative answer means the negative direction. If the positive direction is towards the shop, then the negative direction is towards the school)

Note that the answers to b) and c) above is the same. This is because it we are calculating the same displacement (from the starting point to the end point of Ray's motion) so it is expected that the two answers will be identical.

May walks from her house to the shop to buy some school lunch and then she walks to school



a) What distance did May cover?

b) What is May's displacement if May's house is the reference point and towards the school is the positive direction?

c) What is May's displacement if the school is the reference point and towards the shop is the positive direction?

Speed and velocity

Average speed and velocity

Average speed is the total distance travelled divided by the time interval it took for the motion to occur. It is a scalar. Speed is defined as: [D]

$$S_{ave} = \frac{D}{\Delta t}$$

Average speed (S_{ave}) is measured in meters per second ($m{\cdot}s^{-1}$). Distance (D) is measured in meters (m). The time interval (Δt) is measures in seconds (s). Speed is a scalar meaning that it has magnitude only.

Average velocity is the change in position / displacement ($\Delta\vec{x}$) divided by the time interval it took for the motion to occur. It is a vector. Velocity is defined as: [D]

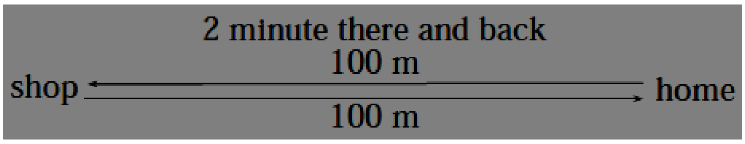
$$\vec{v}_{ave} = \frac{\Delta\vec{x}}{\Delta t}$$

Average velocity ($\vec{v}_{ave}$) is measured in meters per second ($m{\cdot}s^{-1}$). Change in displacement ($\Delta\vec{x}$) is measured in meters (m). The time interval (Δt) is measures in seconds (s). Velocity is a vector meaning that it has both magnitude and direction.

For the same motion, the average speed and the velocity may have different values. The table below summarises the differences between speed and velocity:

Speed	**Velocity**
Scalar	Vector
Always positive	May be positive or negative
Depends on the path taken	Does not depend on the path taken

For example, Sarah walks to the shop to buy some milk. After walking 100 m, she realises that she does not have enough money and goes back home. If it took her two minutes to leave and come back, calculate the following (with the aid of the diagram below):



How long was Sarah out of the house?

What was the distance that Sarah walked?

What was Sarah's displacement?

What was Sarah's average speed?

What was Sarah's average velocity?

<u>Instantaneous speed and velocity</u>

Imagine a person runs a 200 m race and it takes them 30 seconds to complete it. When they take off, they are moving slower than they are moving 20 seconds into the race. While average speed/velocity considers all the different speeds/velocities during a motion and averages them out, instantaneous speed/velocity measures the speed/velocity at a certain point in time. We do not use the symbol Δt for time since we are not considering a time interval. We simply use the symbol t for time.

What is instantaneous speed? [D]

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$S_{inst} = \frac{D}{t}$

What is instantaneous velocity? [D]

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$\vec{v}_{inst} = \frac{\Delta\vec{x}}{t}$

Speed conversions

We need to be able to convert $km.h^{-1}$ to $m.s^{-1}$. To do that, we **multiply by 1000 and divide by 3600**. For example, convert 85 $km.h^{-1}$ into $m.s^{-1}$.

$85\ km.h^{-1} = \frac{85\ km}{1\ h}$

To convert from km to m, we multiply the 85 km by 1000 since there are 1000 m in one km.
To convert from h to s, we multiply the 1 hour by 3600 since there are 3600 s in one hour.

$85\ km.h^{-1} = \frac{85\ km}{1\ h} = \frac{85 \times 1000}{1 \times 3600} = \frac{85000}{3600} = 23.61\ m.s^{-1}$

Or simply:

$85\ km.h^{-1} = 85 \times 1000 \div 3600 = 23.61\ m.s^{-1}$

We also need to be able to convert $m.s^{-1}$ to $km.h^{-1}$. To do that, we **divide by 1000 and multiply by 3600**. For example, convert 15 $m.s^{-1}$ into $km.h^{-1}$:

$15\ m.s^{-1} = \frac{15\ m}{1\ s}$

To convert from m to km, we divide by 1000 (or multiply by 10^{-3}) since there are 1000 m in one km.
Since there are 3600 s in one hour, then 1 s is equal to $\frac{1}{3600}$ of an hour.

$15\ m.s^{-1} = \frac{15\ m}{1\ s} = \frac{15 \times 10^{-3}}{\frac{1}{3600}} = 15 \times 10^{-3} \times 3600 = 54\ km.h^{-1}$

Or simply:

$15\ m.s^{-1} = 15 \div 1000 \times 3600 = 54\ km.h^{-1}$

Convert 60 $km.h^{-1}$ into $m.s^{-1}$:

Convert 17.65 $m.s^{-1}$ into $km.h^{-1}$:

Acceleration

Acceleration is the change velocity ($\Delta\vec{v}$) divided by the time interval it took for the motion to occur. It is a vector. Acceleration is defined as: [D]

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$$

Acceleration ($\vec{a}$) is measured in meters per second squared ($m \cdot s^{-2}$). Change in velocity ($\Delta\vec{v}$) is measured in meters per second ($m.s^{-1}$). The time interval (Δt) is measures in seconds (s). Acceleration is a vector meaning that it has both magnitude and direction.

Like displacement and velocity, acceleration can be positive or negative, but this does not depend only on the direction that the object moves. It also depends on whether the object is speeding up or slowing down:

- If the direction of the motion is positive and the object is speeding up, then + and + will make a +, hence a positive acceleration.
- If the direction of the motion is negative and the object is slowing down, then – and – will make a +, hence a positive acceleration.
- If the direction of the motion is positive and the object is slowing down, then + and – will make a –, hence a negative acceleration.
- If the direction of the motion is negative and the object is speeding up, then – and + will make a –, hence a negative acceleration.

This means that if your final answer for acceleration is positive, it is possible that the motion is in the positive or in the negative direction. If your final answer for acceleration is negative, it is also possible that the motion is in the positive or in the negative direction. You need to look at the context of the question to figure out if the motion is speeding up or slowing down to help you to identify the direction.

We may not use the word deceleration to refer to a slowing down. Acceleration is either positive or negative, and that depends on both the properties mentioned above. Consider the example below:

A car accelerates uniformly from an initial velocity of $2\ m \cdot s^{-1}$ to a final velocity of $10\ m \cdot s^{-1}$ in 8 seconds. Calculate the acceleration of the car during the 8 seconds.

The answer is positive. We know based on the context of the question that the car is speeding up (which is positive) so the direction must be positive. The positive direction is the direction of the motion.

From 8 s to 14 s, the car then slows down uniformly to a final velocity of $4\ m \cdot s^{-1}$. Calculate the acceleration of the car from 8-14s.

The answer is negative. We know based on the context of the question that the car is slowing down (which is negative) so the direction must be positive. The positive direction is the direction of the motion.

Describing motion and motion graphs

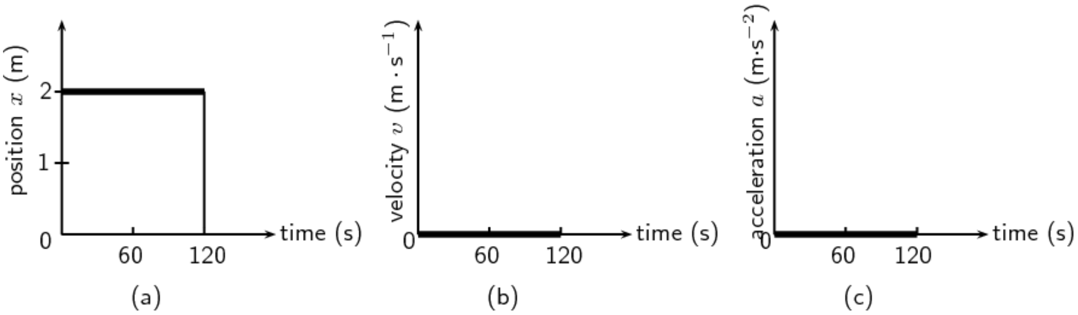
Now that we understand the definitions of distance, displacement, speed, velocity and acceleration, we may use these ideas to describe how an object moves and represent the motion using a graph. We will consider three types of motion:

- Scenario 1: Stationary / the object does not move.
- Scenario 2: Uniform motion / the object moves at a constant velocity / the object moves with a constant change in displacement.
- Scenario 3: Constant acceleration / the object moves with a constant change in velocity.

Scenario 1: Stationary object

The simplest motion that we can come across is that of a stationary object. A stationary object does not move and so its position does not change over time. Tammy waits for her mom to pick her up from school. The stop street is the point of reference, and she is standing 2 m away from the stop street in the positive direction at t = 0 s. After one minute, at t = 60 s, she is still standing 2 m away from the stop street in the positive direction. After two minutes, at t = 120 s, she is still standing 2 m away from the stop street in the positive direction. Her position over the 2 minutes does not change. This means that her displacement is zero. Because her displacement is zero, her velocity is zero. Because her velocity is zero, her acceleration is zero.

We may now draw graphs of position-time ($\vec{x}\ vs\ t$), velocity-time ($\vec{v}\ vs\ t$) and acceleration-time ($\vec{a}\ vs\ t$) for Tammy who was standing 2 m away from the stop street in the positive direction for 120 s. (Time is the independent variable on the x-axis. Displacement is the dependent variable on the y-axis. Because displacement is represented by $\vec{x}$, this may be confusing).



What does graph (a) of ($\vec{x}\ vs\ t$) show?

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What does graph (b) of ($\vec{v}\ vs\ t$) show?

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What does graph (c) of ($\vec{a}\ vs\ t$) show?

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Recall from maths that the gradient, m, of a line can be calculated by dividing the change in the y (dependent variable) by the change in the x (independent variable):

$$m = \frac{\Delta y}{\Delta x}$$

What does the gradient of an $\vec{x}\ vs\ t$ graph represent?

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In the case of constant velocity (scenario 2), the gradient of an $\vec{x}\ vs\ t$ graph represents the ___________________ _______________________.

In the case of a constant change in velocity (scenario 3), the tangent of an $\vec{x}\ vs\ t$ graph represents the ____.

What does the gradient of an $\vec{v}\ vs\ t$ graph represent?

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Because we only deal with constant acceleration, average acceleration and instantaneous acceleration will always be the same.

Let's calculate Tammy's velocity by considering the $\vec{x}\ vs\ t$ graph above:

The flat gradient of an $\vec{x}\ vs\ t$ graph represents zero velocity.

Let's calculate Tammy's acceleration by considering the $\vec{v}\ vs\ t$ graph above:

The flat gradient of a $\vec{v}\ vs\ t$ graph represents zero acceleration.

In summary:

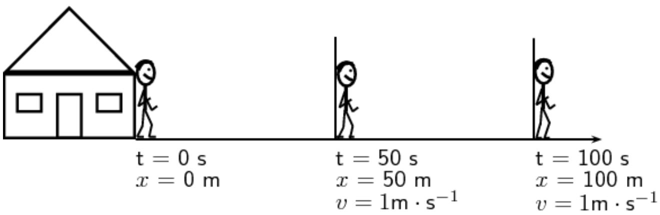
- The gradient of an $\vec{x}\ vs\ t$ graph represents velocity.
- The gradient of a $\vec{v}\ vs\ t$ graph represents acceleration.

Two other important points to note:

- The area under a $\vec{v}\ vs\ t$ graph represents displacement. In Tammy's case above, there is no area to calculate under the $\vec{v}\ vs\ t$ graph, meaning that there was zero displacement.
- The area under an $\vec{a}\ vs\ t$ graph represents velocity. In Tammy's case above, there is no area to calculate under the $\vec{a}\ vs\ t$ graph, meaning that there was zero velocity.

<u>Scenario 2: Uniform motion</u>

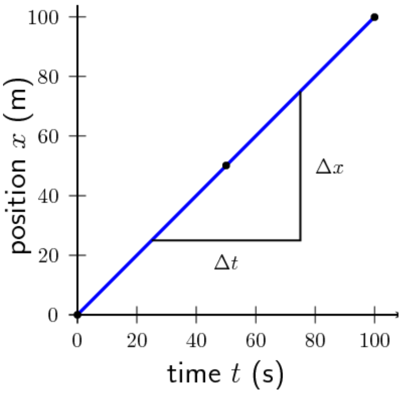
Uniform motion means that the object moves at a constant velocity. This also means that the object moves with a constant change in displacement, so that the position of the object is changing at the same rate. In the morning, Tammy walks from her home to the bus stop, which is 100 m away from her home. It takes her 100 s to walk from her home to the bus stop. A diagram of Tammy's position is shown below:



We may now draw graphs of position-time ($\vec{x}\ vs\ t$), velocity-time ($\vec{v}\ vs\ t$) and acceleration-time ($\vec{a}\ vs\ t$) for Tammy moving at a constant velocity. Let the origin be Tammy's house, and towards the bus stop be the positive direction.

Tammy's change in displacement shows that she walked 1 m in the first second, another meter in the second second, another meter in the third second, and so on. After 50 s, she covered 50 m. Her position increased by one meter every one second.

The position-time ($\vec{x}\ vs\ t$) graph for this motion looks like this:



What does the $\vec{x}\ vs\ t$ graph show?

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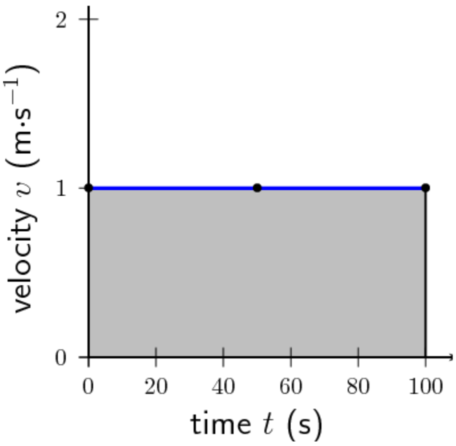
Calculate Tammy's velocity. The gradient of an $\vec{x}\ vs\ t$ graph represents velocity.

Tammy's velocity is positive since she walked in the positive direction. The fact that the $\vec{x}\ vs\ t$ graph has a single gradient means that Tammy's velocity is constant. The fact that the $\vec{x}\ vs\ t$ graph has a positive gradient means that Tammy is walking in the positive direction. Now that we know that Tammy's velocity is 1 m.s^{-1} towards the bus stop for 100 s, we may draw the $\vec{v}\ vs\ t$ graph:



What does the $\vec{v}$ vs t graph show?

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Since Tammy's velocity is constant, is Tammy accelerating?

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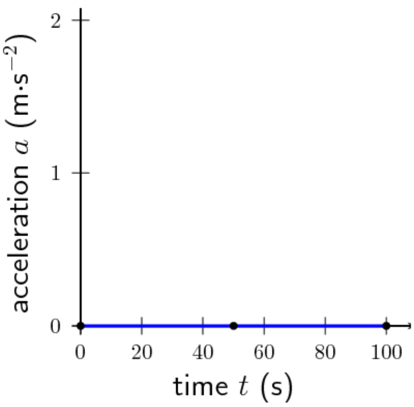
Tammy's acceleration is 0 m.s^{-2} for the 100 s, which is represented by the $\vec{a}$ vs t graph below:



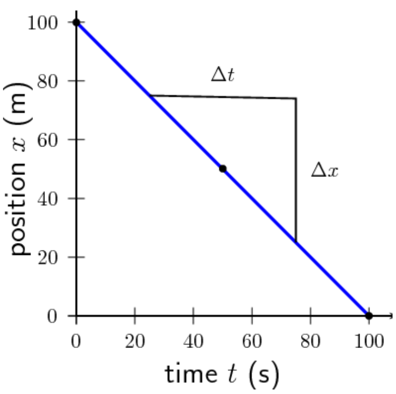
Now let's go back to the $\vec{v}$ vs t graph. Recall that the area under a $\vec{v}$ vs t graph represents displacement. This is because:

$$\vec{v} = \frac{\Delta\vec{x}}{\Delta t}$$

$$\therefore \Delta\vec{x} = \vec{v}\Delta t$$

The formula for the area of a square or rectangle is length × breadth. If you look at the $\vec{v}$ vs t graph above, you will notice that the length is represented by t, and the breadth is represented by $\vec{v}$. Multiplied together, we get $\Delta\vec{x}$, or the change in position. Let's calculate Tammy's displacement from the $\vec{v}$ vs t graph above:

After school, Tammy walks 100 m from the bus stop to her house. It takes her 100s to walk the 100 m back home. The position-time ($\vec{x}$ vs t) graph for this motion looks like this:



What does the $\vec{x}$ vs t graph show?

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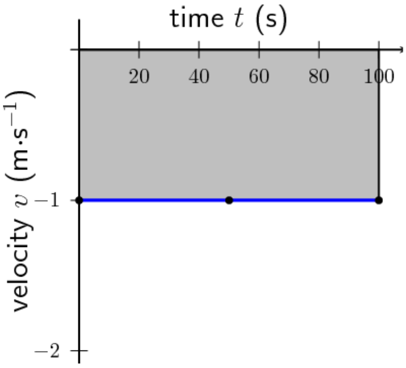
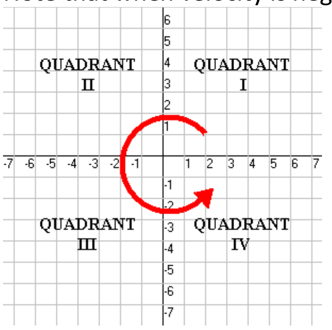
Calculate Tammy's velocity. The gradient of an $\vec{x}$ vs t graph represents velocity.

Tammy's velocity is negative since she walked in the negative direction. The fact that the $\vec{x}$ vs t graph has a single gradient means that Tammy's velocity is constant. The fact that the $\vec{x}$ vs t graph has a negative gradient means that Tammy is walking in the negative direction. Now that we know that Tammy's velocity is 1 $m.s^{-1}$ towards her house, we may draw the $\vec{v}$ vs t graph:



Note that when velocity is negative, it is graphed in the fourth quadrant (IV).



What does the $\vec{v}$ vs t graph show?

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Since Tammy's velocity is constant, is Tammy accelerating?

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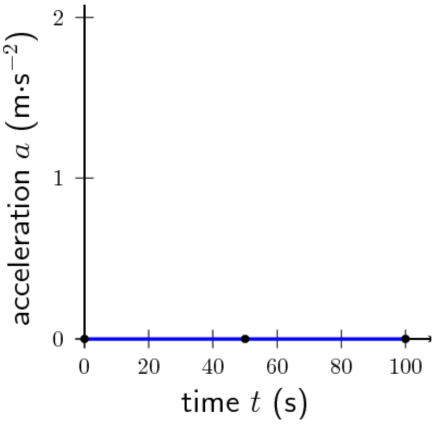
Tammy's acceleration is 0 m.s^{-2} for the 100 s, which is represented by the $\vec{a}$ vs t graph below:



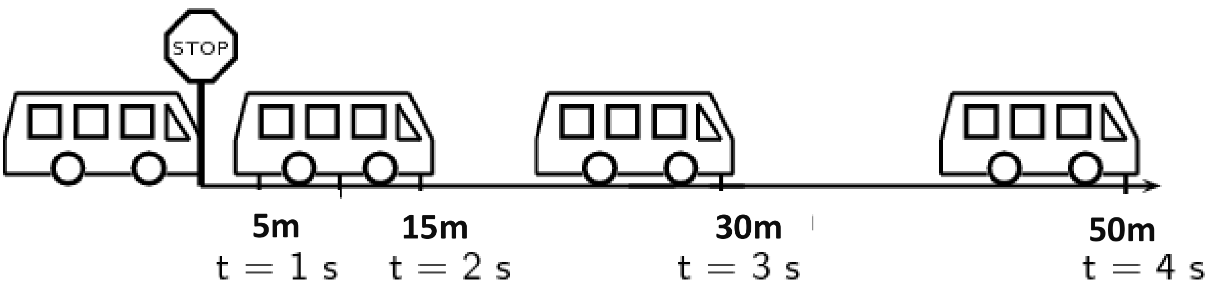
Now let's go back to the $\vec{v}$ vs t graph. Recall that the area under a $\vec{v}$ vs t graph represents displacement. Let's calculate Tammy's displacement from the $\vec{v}$ vs t graph above.

<u>Scenario 3: Constant acceleration</u>

Acceleration is the rate of change of velocity. A constant acceleration means that the velocity changes at a constant rate.

Tammy's mom picks her up from school, and Tammy gets in the car. This is the point of origin. The car accelerates (speed up) in the direction of the motion (positive direction):

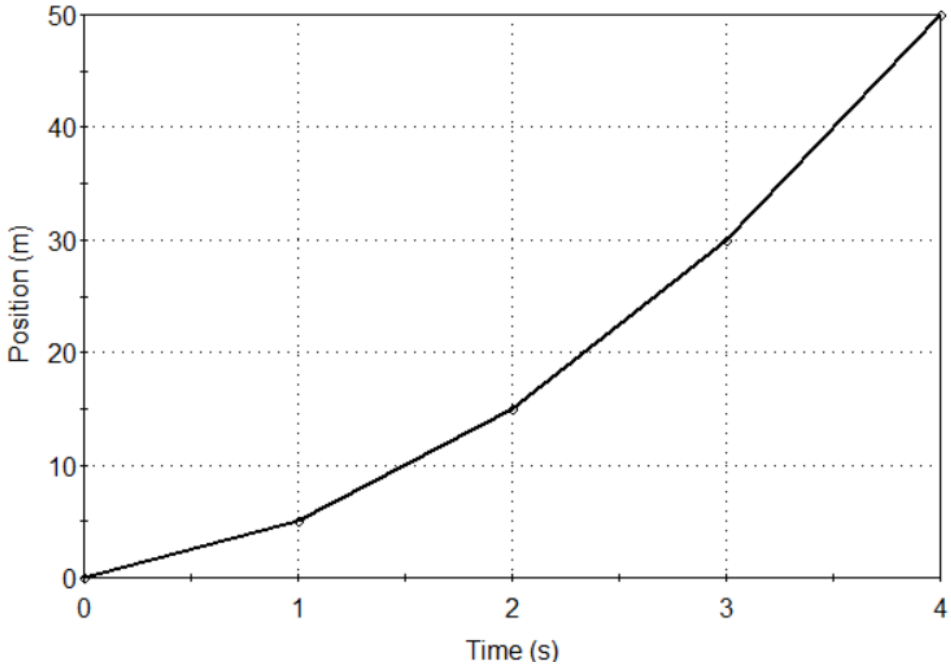
- After 1 s the car covered a distance of 5 m from the origin.
- After 2 s the car covers a distance of 15 m from the origin.
- After 3 s the car covered a distance of 30 m from the origin.
- After 4 s the car covered a distance of 50 m from the origin.



We can graph the $\vec{x}$ vs t graph for this motion based on the information above and the table below:

Time (s)	Position (m)
1	5
2	15
3	30
4	50

The $\vec{x}$ vs t graph for this motion has a curved shape:

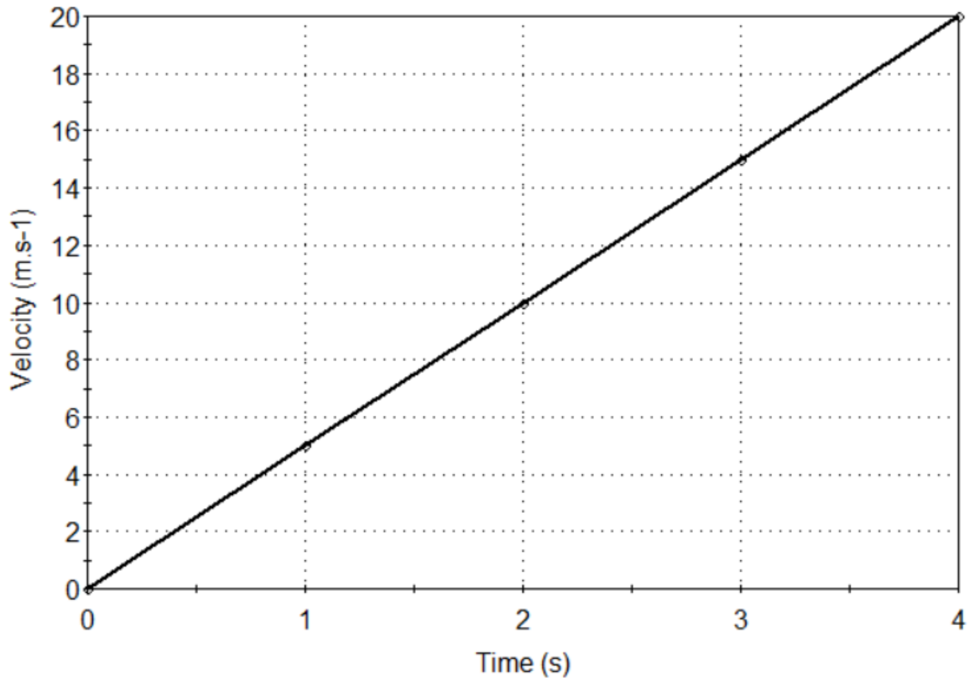


An $\vec{x}$ vs t graph will be curved when the acceleration is constant. When the acceleration is constant, it means that the velocity is not constant, but that there is a constant <u>change</u> in velocity. We therefore need to calculate the instantaneous velocity at each second to graph the $\vec{v}$ vs t graph.

$$\vec{v}(1s) = \frac{\Delta\vec{x}}{\Delta t}$$
$$= \frac{\vec{x}f - \vec{x}i}{tf - ti}$$
$$= \frac{5-0}{1-0}$$
$$= \frac{5}{1}$$
$= 5\ m.s^{-1}$ in the direction of the motion

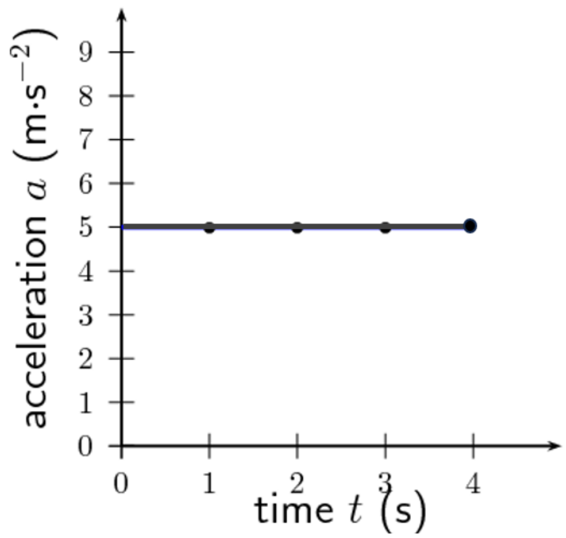
Calculate $\vec{v}(2s)$, $\vec{v}(3s)$ and $\vec{v}(4s)$, then graph the $\vec{v}$ vs t graph.

The $\vec{v}$ vs t graph for this motion is a straight-line graph:



Now we may calculate the acceleration of the motion based on the gradient of the $\vec{v}$ vs t graph. From there, we may graph the $\vec{a}$ vs t graph.

The fact that the $\vec{v}$ vs t graph has a single gradient means that the car's acceleration is constant. The fact that the $\vec{v}$ vs t graph has a positive gradient means that the car is traveling in the positive direction. Note that a positive answer may mean speeding up in the positive direction or slowing down in the negative direction. In this scenario the car is speeding up in the direction of the motion. The car travels for 4 seconds at a constant positive acceleration of 5 m.s^{-2}. The $\vec{a}$ vs t graph is represented below:



Recall that the area under a $\vec{a}\ vs\ t$ graph represents velocity. This is because:

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$$

$$\therefore \Delta\vec{v} = \vec{a}\Delta t$$

The formula for the area of a square or rectangle is length × breadth. If you look at the $\vec{a}\ vs\ t$ graph above, you will notice that the length is represented by t, and the breadth is represented by $\vec{a}$. Multiplied together, we get $\Delta\vec{v}$, or the change in velocity.

Let's calculate the car's velocity after 1 s from the $\vec{a}\ vs\ t$ graph above:

Let's calculate the car's velocity after 4 s from the $\vec{a}\ vs\ t$ graph above:

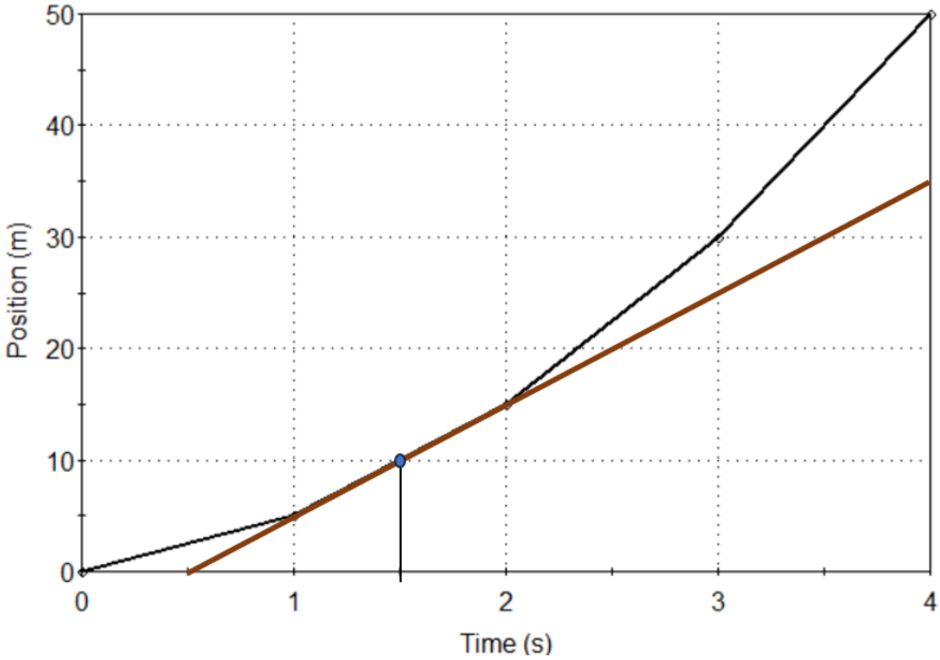
Using a tangent to work out instantaneous velocity from an $\boldsymbol{\vec{x}\ vs\ t}$ t graph

When the velocity is constant, the $\boldsymbol{\vec{x}\ vs\ t}$ graph will have a single gradient that allows us to calculate the velocity throughout the motion. However, when the velocity is not constant, but the acceleration is constant, then the $\boldsymbol{\vec{x}\ vs\ t}$ graph is a parabola. We therefore use a tangent to calculate the instantaneous velocity.

A tangent is a line that touches the curve at one point only. This tangent provides us with a single gradient at that point in time which allows us to calculate velocity at that point in time. Just like in the case of a single gradient graph, we turn the tangent into a right-angled triangle and read off two $\boldsymbol{\vec{x}}$ values and two $\boldsymbol{t}$ values using the formula:

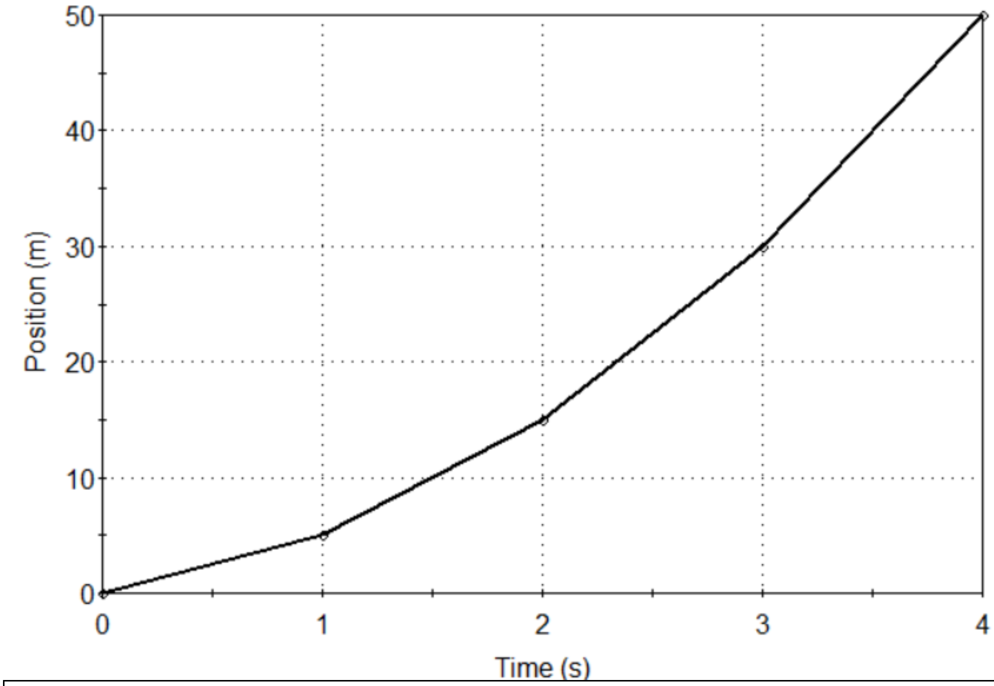
$$\vec{v} = \frac{\Delta\vec{x}}{\Delta t}$$
$$= \frac{\vec{x}f - \vec{x}i}{tf - ti}$$

If you consider "Scenario 3: Constant acceleration" above, we calculated the velocity of the car at 1s, 2s, 3s and 4s. At those time intervals, we had exact position and time values based on the table. But now let's calculate the gradient of the $\boldsymbol{\vec{x}\ vs\ t}$ graph at 1.5 seconds. We need to find the 1.5 s point on the graph. At that point on the curve, we need to draw in a **tangent**. The tangent is a straight line that touches the curve at that point but does not cross it. This line represents the slope of the curve at the exact point where it touches, indicating the instantaneous velocity at that point. We may extend this line as far as we like but it is less confusing to extend the line until it touches the x and the y axes. From there we need to read the values off the graph with accuracy.



$$\vec{v}(1.5s) = \frac{\Delta\vec{x}}{\Delta t}$$
$$= \frac{\vec{x}f - \vec{x}i}{tf - ti}$$
$$= \frac{34-0}{4-0.5}$$
$$= \frac{34}{3.5}$$
$= 9.71\ m.s^{-1}$ in the direction of the motion

Using a tangent, calculate the velocity of the car at 3.5 seconds:



Summary of graphs

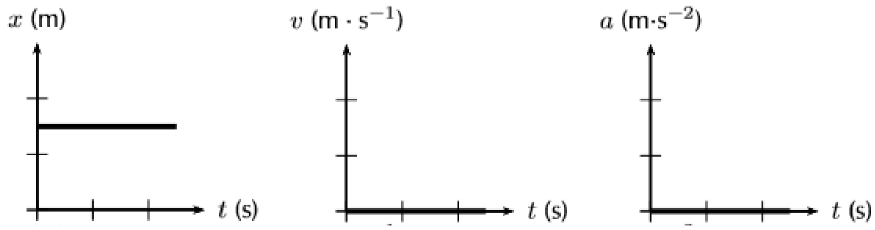
When you describe the motion represented by a graph, you should include the following information:

- Whether the object is at rest, moving at constant velocity or moving at constant acceleration.
- Whether the object is moving in the positive or negative direction.
- For acceleration, state if the object is speeding up or slowing down.
- The duration of the motion (time).

The relationship between graphs of position, velocity and acceleration as functions of time is summarised blow:

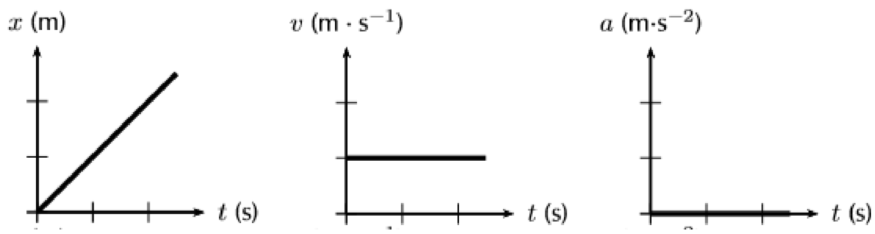
<u>Scenario 1: Stationary object</u>

When an object is stationary, its position does not change as we can see on the $\vec{x}\ vs\ t$ graph. This means that there is zero velocity and zero acceleration.



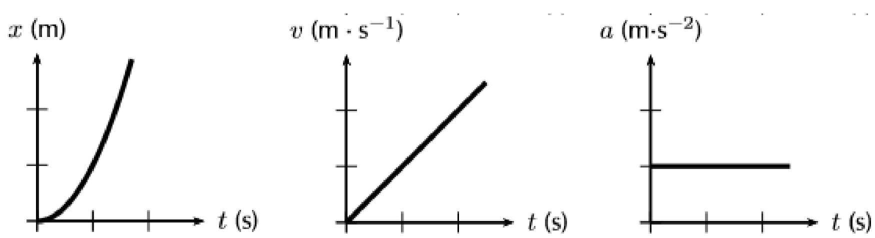
<u>Scenario 2: Uniform motion</u>

When the motion is uniform, it means that the object is moving with a constant velocity. It's position therefore changes by the same amount every second, resulting in a single gradient $\vec{x}\ vs\ t$ graph. The gradient of the $\vec{x}\ vs\ t$ provides us with the velocity value. Because the velocity is not changing, the $\vec{v}\ vs\ t$ graph is a straight line. The area under the $\vec{v}\ vs\ t$ graph represents the change in position, $\Delta\vec{x}$ of the object. Because the object is moving at a constant velocity, there is zero acceleration (no speeding up or slowing down).



<u>Scenario 3: Constant acceleration</u>

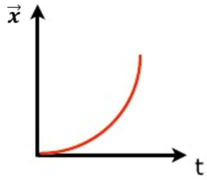
When the object is moving at a constant acceleration, it means that the object is moving with a constant change in velocity. In the graph below, the position is changing every second by an increasing amount since the object is speeding up, resulting in a parabolic $\vec{x}\ vs\ t$ graph. The object's velocity changes by the same amount every second, resulting in a single gradient $\vec{v}\ vs\ t$ graph. The gradient of the $\vec{v}\ vs\ t$ provides us with the acceleration value. The area under the $\vec{v}\ vs\ t$ graph represents the change in position, $\Delta\vec{x}$ of the object. Because the acceleration is not changing, the $\vec{a}\ vs\ t$ **graph** is a straight line. The area under the $\vec{a}\ vs\ t$ graph represents the change in velocity, $\Delta\vec{v}$ of the object.



The above example indicates only one out of four types of $\vec{x}$ $\boldsymbol{vs}$ $\boldsymbol{t}$ graphs for constant acceleration. Let's recap all the potential $\vec{x}$ $\boldsymbol{vs}$ $\boldsymbol{t}$ graphs for constant acceleration:

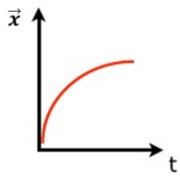
Speeding up in the positive direction (positive acceleration)

In this case, the object is speeding up in the positive direction. We know that the direction is positive since the curve has an upward slope. We know that the object is speeding up since the graph starts off less steep (slower), then becomes more steep (faster). This is a positive acceleration so the $\vec{a}$ $\boldsymbol{vs}$ $\boldsymbol{t}$ graph is positive meaning that it is drawn in the first quadrant.



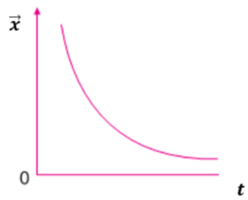
Speeding up in the negative direction (negative acceleration)

In this case, the object is slowing down in the positive direction. We know that the direction is positive since the curve has an upward slope. We know that the object is slowing down since the graph starts off more steep (faster), then becomes less steep (slower). This is a negative acceleration so the $\vec{a}$ $\boldsymbol{vs}$ $\boldsymbol{t}$ graph is negative meaning that it is drawn in the fourth quadrant.



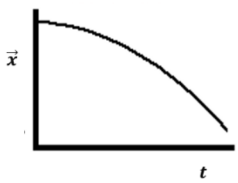
Slowing down in the negative direction (positive acceleration)

In this case, the object is slowing down in the negative direction. We know that the direction is negative since the curve has a downward slope. We know that the object is slowing down since the graph starts off more steep (faster), then becomes less steep (slower). This is a positive acceleration so the $\vec{a}$ $\boldsymbol{vs}$ $\boldsymbol{t}$ graph is positive meaning that it is drawn in the first quadrant.



Speeding up in the negative direction (negative acceleration)

In this case, the object is speeding up in the negative direction. We know that the direction is negative since the curve has a downward slope. We know that the object is speeding up since the graph starts off less steep (slower), then becomes more steep (faster). This is a negative acceleration so the $\vec{a}$ $\boldsymbol{vs}$ $\boldsymbol{t}$ graph is negative meaning that it is drawn in the fourth quadrant.

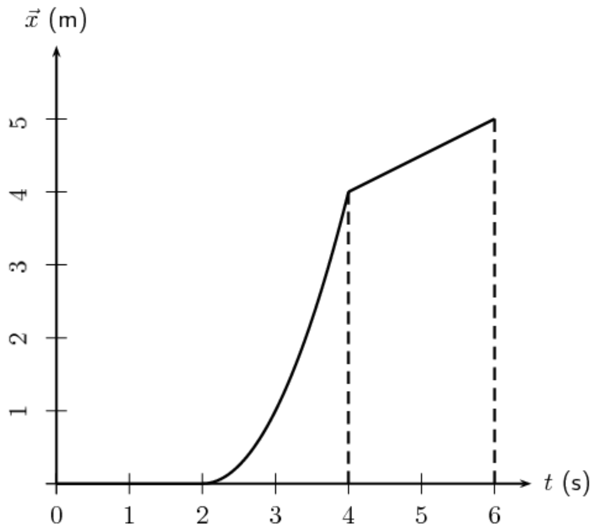


Examples

The $\vec{x}$ $\boldsymbol{vs}$ $\boldsymbol{t}$ graph for the motion of a car is given below. Let the direction of the motion be the positive direction.

a) Describe the motion of the car at 0-2 s, 2-4 s and 4-6 s.
b) Calculate the velocity of the car from 4-6 s from the $\vec{x}$ $\boldsymbol{vs}$ $\boldsymbol{t}$ graph.
c) Draw the corresponding $\vec{v}$ $\boldsymbol{vs}$ $\boldsymbol{t}$.
d) Calculate the acceleration of the car from 2-4 s from the $\vec{v}$ $\boldsymbol{vs}$ $\boldsymbol{t}$ graph.
e) Draw the corresponding $\vec{a}$ $\boldsymbol{vs}$ $\boldsymbol{t}$ graphs.
f) Calculate the displacement of the car from the $\vec{v}$ $\boldsymbol{vs}$ $\boldsymbol{t}$ graph.

(Note that unless otherwise stated, we assume that the origin at is at the x-y intercept)



a) Motion at 0-2 s:

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Motion at 2-4 s:

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Motion at 4-6 s:

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b) Calculate the velocity of the car from 4-6 s:

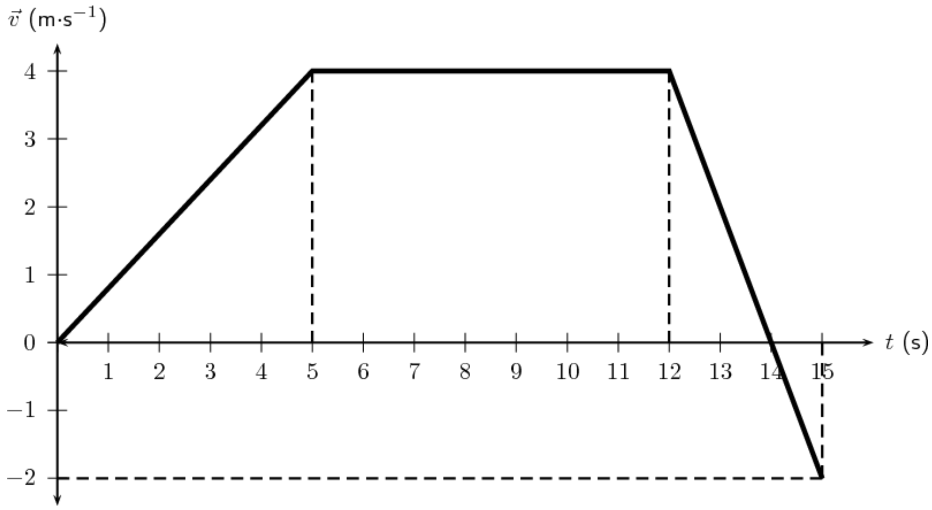
c) Corresponding $\vec{v}$ $\boldsymbol{vs}$ $\boldsymbol{t}$ graph:

d) Calculate the acceleration of the car from 2-4 s:

e) Corresponding $\vec{\boldsymbol{a}}$ $\boldsymbol{vs}$ $\boldsymbol{t}$ graph:

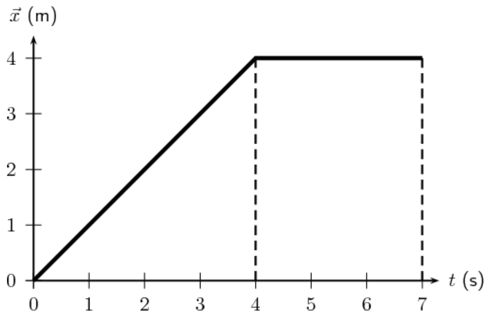
f) The displacement of the car from the $\vec{v}$ ***vs t*** graph:

The $\vec{v}$ ***vs t*** graph of a truck is plotted below. Calculate the distance and displacement of the truck after 15 s. Let the direction of the motion be the positive direction.



The $\vec{x}$ *vs t* graph below describes the motion of an athlete. Let West be the positive direction.

a) Calculate the velocity of the athlete during the first 4 seconds.
b) What is the velocity of the athlete from 4-7 s?
c) Describe the motion of the athlete.



a) The velocity of the athlete during the first 4 seconds:

b) the velocity of the athlete from 4-7 s:

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c) Describe the motion of the athlete:

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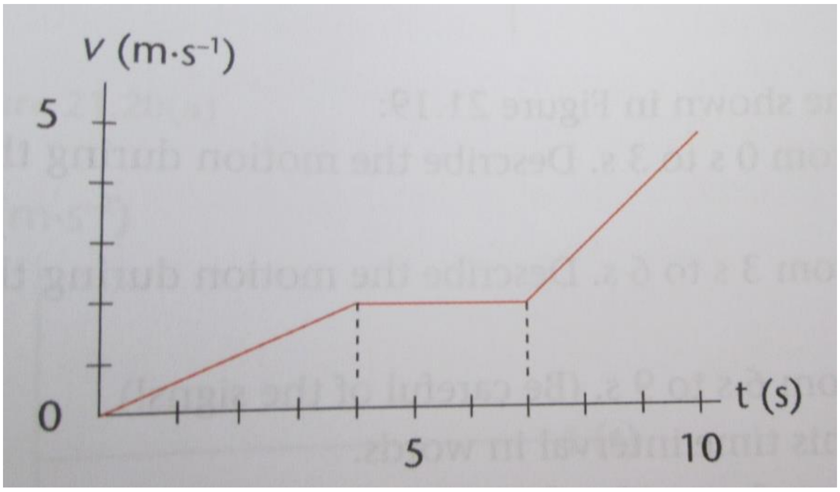
For the motion represented by the $\vec{v}$ *vs t* graph below, calculate how far the object travelled in the first 4 s, from 4-7 s and from 7-10 s. Draw the corresponding $\vec{x}$ *vs t* graph for the motion. Let the direction of the motion be the positive direction.

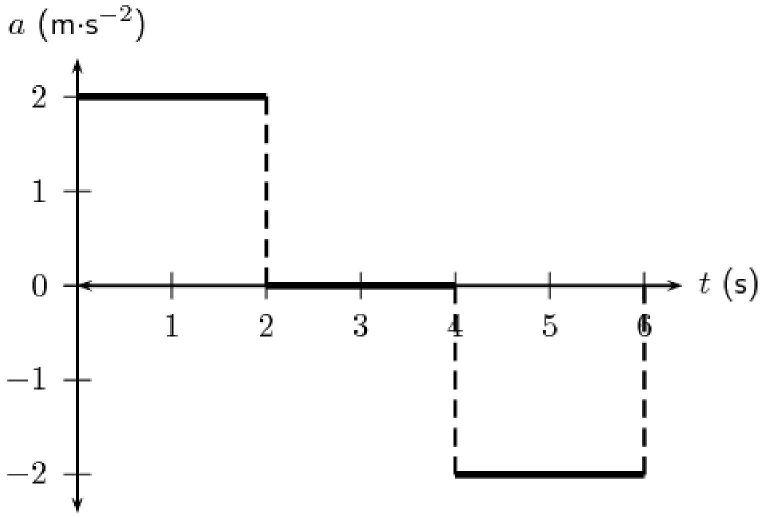


How far the object travelled in the first 4 s, from 4-7 s and from 7-10 s:

Corresponding $\vec{x}$ $\boldsymbol{vs}$ $\boldsymbol{t}$ graph:

The $\vec{a}$ $\boldsymbol{vs}$ $\boldsymbol{t}$ graph for a car traveling East given below. Let East be the positive direction.

a) Calculate the velocity of the car from 0-2 s, 2-4 s and 4-6 s.
b) Draw the corresponding $\vec{v}$ $\boldsymbol{vs}$ $\boldsymbol{t}$ graph.
c) Describe the motion of the car from the perspective of the $\vec{v}$ $\boldsymbol{vs}$ $\boldsymbol{t}$ graph.
d) Describe the motion of the car from the perspective of the $\vec{a}$ $\boldsymbol{vs}$ $\boldsymbol{t}$ graph.



a) The velocity of the car from 0-2 s, 2-4 s and 4-6 s:

b) The corresponding $\vec{v}$ $\boldsymbol{vs}$ $\boldsymbol{t}$ graph:

c) From the perspective of the $\vec{v}$ vs t graph:

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d) From the perspective of the $\vec{a}$ vs t graph:

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